College Admissions with Entrance Exams: Centralized versus Decentralized

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November 7, MiP, Lisbon
We study a **college admissions problem** in which

- colleges accept students by ranking students’ efforts or scores in entrance exams, and
- students hold private information regarding their ability level that affects the cost of efforts.
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- colleges accept students by ranking students’ efforts or scores in entrance exams, and
- students hold private information regarding their ability level that affects the cost of efforts.

Two mechanisms:

1. **Centralized college admissions (CCA)**
   - Admission decision is through a clearinghouse.

2. **Decentralized college admissions (DCA)**
   - Admission decision is independently made by each college among its applicants.
Motivation

Every year, millions of prospective college students apply for admissions to colleges.

- Admissions are mostly based on the results of exam scores.
- Students put a lot of effort to do well in the exams, and are heterogeneous in terms of ability.

1. In some countries, the process is very centralized.
   - China, Greece, Korea, Taiwan, Turkey

2. In some other countries the process is decentralized.
   - Japan: For each student, only one application is made to public universities.

3. United States falls in between the two extreme cases.
The Model: $(C, S, (q_1, q_2), (v_1, v_2), F)$

1. $C = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
The Model: \((C, S, (q_1, q_2), (v_1, v_2), F)\)

1. \(C = \{1, 2\}\): set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
2. \(S\): a finite set of students.

Each student's ability is drawn i.i.d. from \([0, 1]\) according to the distribution \(F\). Each student's ability is private information.
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2. \(S\): a finite set of students.
3. \((q_1, q_2)\): \(q_C \geq 1\) is the capacity of college \(C \in C\).
   - Assume \(q_1 + q_2 \leq |S|\).
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   - \(v_2 > v_1 > 0\) (attending no college).
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   - Each student’s ability is private information.

6. Type \(a\)’s utility from making effort \(e = \)
   \[
   \begin{cases} 
   v_C - \frac{e}{a} & \text{if she is assigned to college } C, \\
   -\frac{e}{a} & \text{if she is assigned to no college}.
   \end{cases}
   \]
Centralized College Admissions (CCA)

The game under CCA

- Each student $s$ simultaneously makes an effort $e_s$.
- Given efforts $(e_s)_{s \in S}$ of students,
  - Students with the top $q_2$ efforts are assigned to college 2,
  - Students with the top $(q_2 + 1)$ to $(q_1 + q_2)$ are assigned to college 1,
  - The rest of the students are not assigned to any college.

Remark: Note that this is a contest (or an all-pay auction) under incomplete information (Moldovanu and Sela, AER, 2001).
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Remark

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Equilibrium under CCA

**Proposition 1 (Equilibrium in CCA)**

In the centralized college admission mechanism, there is a unique symmetric equilibrium $\beta^C$ such that for each $a \in [0, 1]$, 

$$
\beta^C(a) = \int_0^a \{ f_{n-q_2,n-1}(x) v_2 + (f_{n-q_1-q_2,n-1}(x) - f_{n-q_2,n-1}(x)) v_1 \} \, dx.
$$
The Game under DCA

- Each student \( s \) chooses one college \( C_s \) and an effort \( e_s \) simultaneously.
- Given college choices \( (C_s)_{s \in S} \) and efforts \( (e_s)_{s \in S} \) of students,
  - Each college \( C \) admits students with the top \( q_C \) effort levels among its set of applicants \( \{s \in S \mid C_s = C\} \).
  - The rest of the students are not assigned to any college.
Decentralized College Admissions (DCA)

The Game under DCA

- Each student $s$ chooses one college $C_s$ and an effort $e_s$ simultaneously.
- Given college choices $(C_s)_{s \in S}$ and efforts $(e_s)_{s \in S}$ of students,
  - Each college $C$ admits students with the top $q_C$ effort levels among its set of applicants ($\{s \in S \mid C_s = C\}$).
  - The rest of the students are not assigned to any college.

Remark

This is the first model of competing contests where the players sort themselves into different contests.
Type of Symmetric Strategy for Equilibrium

Solve for a symmetric Bayesian Nash equilibrium \((\gamma, \beta^D; c)\) where \(c\) is a cutoff,

1. A student with low ability \(a \in [0, c]\)
   - chooses college 1 with probability \(\gamma(a)\),
   - makes effort \(\beta^D(a)\).

2. A student with high ability \(a \in [c, 1]\),
   - chooses college 2 for sure,
   - makes effort \(\beta^D(a)\).
Comparisons

Proposition 2

Low ability students prefer DCA to CCA if and only if $n > q_1 + q_2$.

Proposition 3 (Single crossing property)

Let $c$ be the equilibrium cutoff.

1. If $EU^C(a) \geq EU^D(a)$ for some $a > c$, then $EU^C(a') > EU^D(a')$ for all $a' > a$, and
2. if $EU^C(a) < EU^D(a)$ for some $a > c$, then $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$.
2 colleges
12 students
The distribution $F$ is uniform on the ability space $[0, 100]$.
Students received an endowment of 2,200 points. Exchange rate was 0.5 cents in Euro per point.
Design of the Experiment (1)

- 2 colleges
- 12 students
- The distribution $F$ is uniform on the ability space $[0, 100]$. 
- Students received an endowment of 2,200 points. Exchange rate was 0.5 cents in Euro per point.

Five different markets

<table>
<thead>
<tr>
<th>$q_2$</th>
<th>$q_1$</th>
<th>$v_2$</th>
<th>$v_1$</th>
<th>Theoretical comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 1</td>
<td>6</td>
<td>6</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CCA</td>
</tr>
<tr>
<td>M 2</td>
<td>2</td>
<td>2</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>DCA</td>
</tr>
<tr>
<td>M 3</td>
<td>2</td>
<td>8</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>depends; DCA in exp</td>
</tr>
<tr>
<td>M 4</td>
<td>3</td>
<td>9</td>
<td>2000</td>
<td>1800</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CCA</td>
</tr>
<tr>
<td>M 5</td>
<td>9</td>
<td>1</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>no diff. in exp</td>
</tr>
</tbody>
</table>

Pred. utility is higher | Pred. effort is higher
depends | no diff. in exp
no diff. in exp | CCA
no diff. in exp | DCA
Between-subjects design.

- Each session with 24 subjects was either assigned to the treatment of CCA or DCA.
- 24 subjects were split into two matching groups for the entire session.
- Subjects played 15 rounds with one market per round by blocks of five markets.
- Each of the five markets was played three times by every subject.
- Abilities were drawn anew in every round.
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- Each of the five markets was played three times by every subject.
- Abilities were drawn anew in every round.

3 sessions, i.e., 6 independent groups, for each of CCA and DCA.

Experimental economics lab at Technical University Berlin.

Use z-tree (Fischbacher, 2007).

Sessions lasted about 90 minutes; average earnings per subject were EUR 14.10.
Result 1

- In markets 1 and 4, where \( n = q_1 + q_2 \), the average utility of all students in CCA is significantly higher than in DCA.
- In markets 2 and 3, the average utility of all students in DCA is not higher than in CCA, in contrast to the theoretical prediction.
- In market 5, there is no significant difference both in theory and the data.
Treatment Comparison: Expected Utility

Result 1

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<table>
<thead>
<tr>
<th>Market</th>
<th>$q_2$</th>
<th>$q_1$</th>
<th>$v_2$</th>
<th>$v_1$</th>
<th>Average Utility</th>
<th>CCA</th>
<th>DCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>2000</td>
<td>1000</td>
<td>CCA is higher</td>
<td>1223</td>
<td>1021</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2000</td>
<td>1000</td>
<td>DCA is higher</td>
<td>111</td>
<td>86</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>8</td>
<td>2000</td>
<td>1000</td>
<td>DCA is higher</td>
<td>603</td>
<td>576</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>9</td>
<td>2000</td>
<td>1800</td>
<td>CCA is higher</td>
<td>1058</td>
<td>747</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
<td>2000</td>
<td>1000</td>
<td>no difference</td>
<td>1183</td>
<td>1160</td>
</tr>
</tbody>
</table>

Where p-values are for the two-sided Wilcoxon rank-sum test for the equality of distributions of the number of points earned in the two systems.

Presenter: Rustamdan Hakimov
Treatment Comparison: Efforts

Result 2

- In markets 1 and 4, where \( n = q_1 + q_2 \), the average effort level of all students in DCA is significantly higher than in CCA.
- In market 3, the average effort level of all students in CCA is not significantly higher than in DCA.
- In markets 2 and 5, there is no significant difference in effort both in theory and in the data.

<table>
<thead>
<tr>
<th>Market</th>
<th>CCA</th>
<th>DCA</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>8</td>
<td>0.42</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>9</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Where p-values are for the two-sided Wilcoxon rank-sum test for the equality of distributions of the number of points earned in the two systems.
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<table>
<thead>
<tr>
<th>Market</th>
<th>( q_2 )</th>
<th>( q_1 )</th>
<th>( v_2 )</th>
<th>( v_1 )</th>
<th>Average Effort</th>
<th>CCA</th>
<th>DCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>2000</td>
<td>1000</td>
<td>DCA is higher</td>
<td>276</td>
<td>362</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2000</td>
<td>1000</td>
<td>no difference</td>
<td>389</td>
<td>410</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>8</td>
<td>2000</td>
<td>2000</td>
<td>CCA is higher</td>
<td>397</td>
<td>354</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>9</td>
<td>2000</td>
<td>1800</td>
<td>DCA is higher</td>
<td>191</td>
<td>340</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
<td>2000</td>
<td>1000</td>
<td>no difference</td>
<td>400</td>
<td>395</td>
</tr>
</tbody>
</table>

Where p-values are for the two-sided Wilcoxon rank-sum test for the equality of distributions of the number of points earned in the two systems.
Consider markets with $q_1 + q_2 < n$.

- In markets 2 and 3
  - the average utility of low-ability students is significantly higher in DCA as in Proposition 2,
  - the average utility of high-ability students is significantly higher in CCA as in Proposition 3.

- In market 5, there is no difference.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient (Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% ability quantiles</td>
<td>49.008*** (8.069)</td>
</tr>
<tr>
<td>1st quantile in DCA</td>
<td>98.812 (83.255)</td>
</tr>
<tr>
<td>2nd quantile in DCA</td>
<td>294.889*** (76.675)</td>
</tr>
<tr>
<td>3rd quantile in DCA</td>
<td>234.895*** (73.484)</td>
</tr>
<tr>
<td>4th quantile in DCA</td>
<td>57.848 (86.449)</td>
</tr>
<tr>
<td>5th quantile in DCA</td>
<td>-79.696 (93.920)</td>
</tr>
<tr>
<td>6th quantile in DCA</td>
<td>-60.945 (92.340)</td>
</tr>
<tr>
<td>7th quantile in DCA</td>
<td>-278.143*** (91.047)</td>
</tr>
<tr>
<td>8th quantile in DCA</td>
<td>-103.370 (112.019)</td>
</tr>
<tr>
<td>9th quantile in DCA</td>
<td>-190.702 (118.914)</td>
</tr>
<tr>
<td>10th quantile in DCA</td>
<td>-186.753** (110.123)</td>
</tr>
<tr>
<td>Intercept</td>
<td>80.770* (45.231)</td>
</tr>
</tbody>
</table>

| N                             | 864                     |
| R²                            | 0.047                   |
| F (11,852)                    | 3.819                   |
Result 4

The tendency for overexertion of efforts can be observed. However, the equilibrium efforts have significant predictive power for the effort chosen.
Result 4

The tendency for overexertion of efforts can be observed. However, the equilibrium efforts have significant predictive power for the effort chosen.

<table>
<thead>
<tr>
<th></th>
<th>Mean efforts</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lab</td>
<td>equilibrium</td>
</tr>
<tr>
<td>CCA Market 1</td>
<td>276</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 2</td>
<td>389</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 3</td>
<td>397</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 4</td>
<td>191</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 5</td>
<td>400</td>
<td>&gt;</td>
</tr>
<tr>
<td>DCA Market 1</td>
<td>362</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 2</td>
<td>410</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 3</td>
<td>354</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 4</td>
<td>340</td>
<td>&gt;</td>
</tr>
<tr>
<td>Market 5</td>
<td>395</td>
<td>&gt;</td>
</tr>
</tbody>
</table>
Result 5

Students above the equilibrium cutoff choose college 2 significantly more often than students below the cutoff. Across all markets, the equilibrium predictions regarding the probability of choosing the good college have significant predictive power for the subjects’ choices.
Point Predictions Regarding Individual Behavior: Choice of Colleges in DCA

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<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market 1</td>
<td>50</td>
<td>13%</td>
<td>33%</td>
<td>85%</td>
</tr>
<tr>
<td>Market 2</td>
<td>85.5</td>
<td>43%</td>
<td>51%</td>
<td>92%</td>
</tr>
<tr>
<td>Market 3</td>
<td>85.5</td>
<td>15%</td>
<td>27%</td>
<td>68%</td>
</tr>
<tr>
<td>Market 4</td>
<td>89.5</td>
<td>16%</td>
<td>17%</td>
<td>42%</td>
</tr>
<tr>
<td>Market 5</td>
<td>23.5</td>
<td>51%</td>
<td>64%</td>
<td>91%</td>
</tr>
</tbody>
</table>

Presenter: Rustamdjan Hakimov  
College Admissions with Entrance Exams  
November 7, MiP, Lisbon
Table 8: Choice of the good college 2 in DCA

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient (Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium probability of choosing the good college</td>
<td>1.684*** (0.106)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.79*** (0.079)</td>
</tr>
<tr>
<td>N</td>
<td>1080</td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.177</td>
</tr>
</tbody>
</table>
Conclusion

- We theoretically and experimentally study the college admissions process in which
  1. colleges accept students by considering students’ exam scores,
  2. students have differing abilities which are their private information, and costs of effort for the exams are inversely related to ability levels.

- We focus on the two extreme policies:
  1. the centralized college admissions (CCA): students can freely and costlessly apply to all colleges,
  2. the decentralized college admissions (DCA): students can only apply to one college.

- Theory and experiment show **consistently** that:
  - low-ability students prefer DCA, and high-ability students prefer CCA.

- Theory and experiment **differ** with respect to:
  - overexertion of efforts in experiments, especially for DCA.
Equilibrium Predictions for Markets 3 and 4

- **Market 3**
  - Expected utility in equilibrium
  - Equilibrium effort

- **Market 4**
  - Expected utility in equilibrium
  - Equilibrium effort

**Legend**:
- Centralized
- Decentralized

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Equilibrium Predictions for Market 5

![Graphs showing expected utility in equilibrium for Market 5]

Back to [here](#)