The Common Application and the DA Algorithm in School Assignment

Christopher Avery
Cara Nickolaus
Parag Pathak
June, 2015

Overview and Motivation

- Many districts in the US now use matching algorithms to match students and schools.
- There is growing evidence that preferred assignments yield better long-run outcomes.
  (cf: Deming, Hastings, and co-authors).
- But assignment systems may subtly favor advantaged/sophisticated students.

The Common Application

- Many colleges say that the Common Application promotes diversity:

  "We are leveling the playing fields,"

  William Fitzsimmons, Harvard Admissions Director

- Most studies find an increase in applications as a result of the Common Application (Ehrenberg et al 2007, Smith 2011, Klasik 2012, Howell et al. 2014),

Outline of Talk

1. Equivalence of Deferred Acceptance Algorithm to other assignment schemes.

2. Equilibrium results with endogenous application choices.

3. Empirical results from Chicago Public Schools about the effect of the Common Application.

Section 1: Assignment Algorithms

Compare Deferred Acceptance Algorithm to

1. Random Priority rule
2. Common Application system

Key caveat: Schools are not allowed to submit preferences over students.

The Common Application is valuable when schools have strong information about match quality.

Related Literature on Matching

- Many papers on the properties of algorithms.

  Notable papers by Abdulkadiroglu, Budish, Cantillon, Che, Miralles, Pathak, Roth, Sonmez and others.

- Random Priority vs. Probabilistic Serial

  Boglmolnaia and Moulin (2001),

  Kojima and Manea (2008), Kesten (2009),

  Che and Kojima (2010).

The DA Algorithm and School Choice

- Consider two versions of the Deferred Acceptance Algorithm where schools submit random ordinal rankings of students

- 1. All schools submit the same ordinal ranking.
  2. Schools submit independent rankings.

Motivating Example

- Suppose that there are two schools **A** and **B**.
- Each school enrolls 1/3 of all students.
- P(Prefer **A** to **B**) = 3/4; P(Prefer **B** to **A**) = 1/4.

In Round 1 of the DA algorithm,

each school offers to 1/3 of all students.

Results of the Example with Identical School Rankings of Students

In Round 1,

1/3 of students get two offers.

9/36 of students choose **A**, 3/36 choose **B**.

In subsequent rounds, additional students get offers from both schools.

School **A** fills when it offers to 4/9 of students.

School **B** fills when it offers to 2/3 of students.

Results of the Example with Independent Rankings of Students

In Round 1,

2/9 offered by **A**, 2/9 offered by **B**,

1/9 have offers from both.

Eventual solution: $\mathbf{P_A} = .377$, $\mathbf{P_B} = .465$.

Only 17.5% are admitted to both schools and so the assignment is not pairwise stable.

Observations from the Example

- The two versions of the algorithm are analogous to different application rules.

  Identical ranking ~ *Random Priority*

  Independent rankings ~ Common App

- Use of single ranking by schools maximizes opportunity for revealed preference sorting.

Base Model

- Unit measure of applicants.
- **S** Schools, capacity $\mathbf{k_j}$ for school **j**.

  $\mathbf{k_1 + k_2 + \ldots + k_S} < 1.$
- Students have strict preference rankings

  with iid draws from known distribution over possible rank orders for schools 1, 2**,** … **S**.
- Each student prefers assignment to any of schools 1, 2, …, **S** to no assignment.

Formal Results

Proposition 1: *If all schools share a common ranking order of students, the school-proposing DA algorithm and the student-proposing DA algorithm produce the same school assignments as "Random Priority" using that ranking order.*

Proof: By contradiction.

No student who prefers B would be assigned to A under either system …

Existence and Uniqueness

Definition: A Common Application equilibrium consists of admission probabilities ($\mathbf{p_1}$, …, $\mathbf{p_S}$) such that expected enrollment at school **j** is $\mathbf{k_j}$ given independent admissions decisions.

Proposition 2: *There is a unique Common Application equilibrium.*

Proof: By induction and contradiction.

DA and the Common Application

Proposition 3: *If schools submit independent random rankings of students, the school-proposing and student-proposing DA algorithms match students to schools with the same probabilities as the Common Application.*

Intuition for Proof: Each school makes offers to top ranked candidates on its list in a DA algorithm, so we can convert these offers into independent "admission probabilities".

Efficiency Comparisons

- With two schools, the distribution of ordinal rankings of school assignments with Random Priority stochastically dominates the distribution from the Common Application.

- With any number of schools, Random Priority produces pairwise stable outcomes, but Common Application does not.

Section 2: Nash Equilibrium

Compare Nash equilibrium outcomes for

Common Application

"Decentralized" Application systems.

These comparisons will generate hypotheses for empirical tests.

Nash Equilibrium Model

- There are **S** schools and **N** students.
  Each school has capacity **K**.
  Student **i** has utility values ($\mathbf{u_{i1}}$, $\mathbf{u_{i2}}$, ..., $\mathbf{u_{iS}}$).
  Schools are identical ex ante.

<u>Decentralized Rule</u>: Cost **C** > 0 per application.

<u>Common Rule</u>: Cost **C** > 0 to apply to all schools.

Relevant Literature re: Nash Eq.

- The Common Application simplifies decisions.

  Chade, Lewis, and Smith (2014)

- But, it reduces preference signaling and revealed preference matching.

  Avery and Levin (2010)

- It may also increase congestion and induce non-threshold admission rules.

  Lien (2009), Che and Koh (2014)

Existence of Equilibrium

The existence of a unique (uniform) equilibrium admission probability for the Common Application is straightforward.

An increase in admission probability promotes additional applications / enrollment.

This argument fails for the decentralized case!

Numerical Example 2

- 8**N** students and 2 schools.
- Each school wishes to enroll 3**N** students.
- Half of the students have utility values (8,4); the other half have utility values (4, 8).
- If each student applies to 1 school, **p*** = 3/4.

  If each student applies to 2 schools, **p*** = 1/2.
- Multiple decentralized equilibria if $3/4 \leq$ **C** ≤ 1.

Equilibrium Results for Example 2

Cost C	**Decentralized**	**Common App**
C < 3/4	Two Applications	Two Applications
3/4 < **C** < 1	Multiple Eqm. One or Two Apps	Two Applications
1 < **C** < 5	One Application	Two Applications

Nash Equilibrium re: Applications

Proposition 4: *Each school receives more applications and has a lower admissions rate in a Common Application equilibrium than in any (symmetric) decentralized equilibrium.*

Intuition for Proof: For given admission probability, more students will apply with Common than decentralized rule – this drives the equilibrium admissions rate down.

Nash Equilibrium re: Matching

Proposition 5: *The ordinal rankings of student assignments in any decentralized equilibrium stochastically dominate those produced in the unique Common Application equilibrium.*

Intuition for Proof: Since $\mathbf{p}_D > \mathbf{p}_C$, there is less overlap of admission to 1st choice / 2nd choice school with Common Application … so admission must be more spread out across top **S** choices with Common than decentralized rule.

Results for Disadvantaged Types

Suppose that there are two types of applicants – insiders and outsiders, where application costs are lower for insiders than for outsiders.

If insiders apply to all schools, then introducing the Common Application must improve placements for outsiders. But otherwise, the Common App has ambiguous results.

Section 3: Empirical Analysis

- Chicago Public Schools (CPS) has a mixed system for assigning students to kindergarten and also to high school.
- Some spots in schools are assigned through a formal application system; others are assigned by neighborhood of residence.
- CPS changed from a decentralized system to a Common Application system in 2010.

Data and Empirical Hypotheses

- We have five years of application and enrollment data for CPS students:

    Decentralized: 2009-10

    Common: 2010-11 through 2013-14

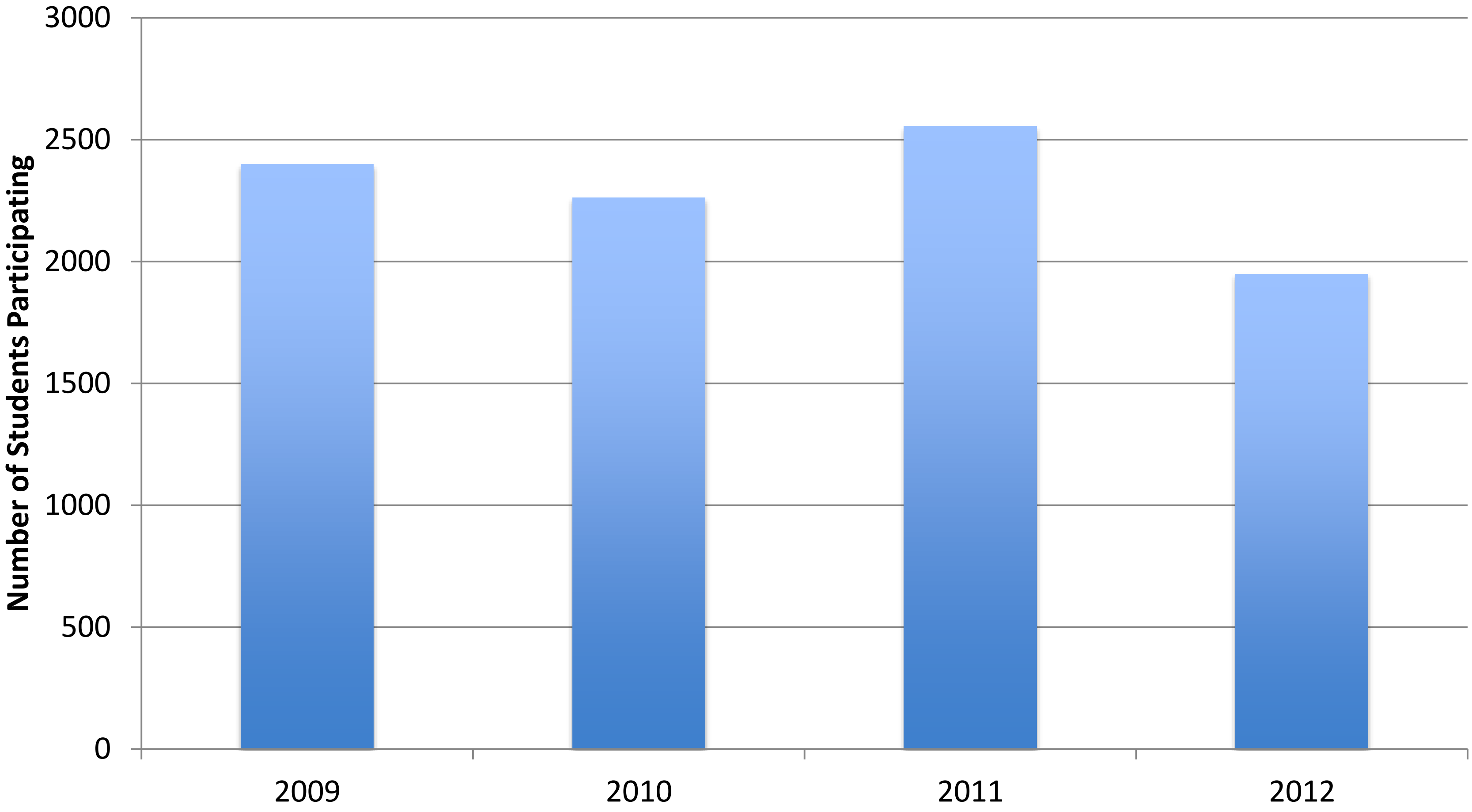
H1: Applications per school increased.

H2: Admission rates decreased after 2009-10;

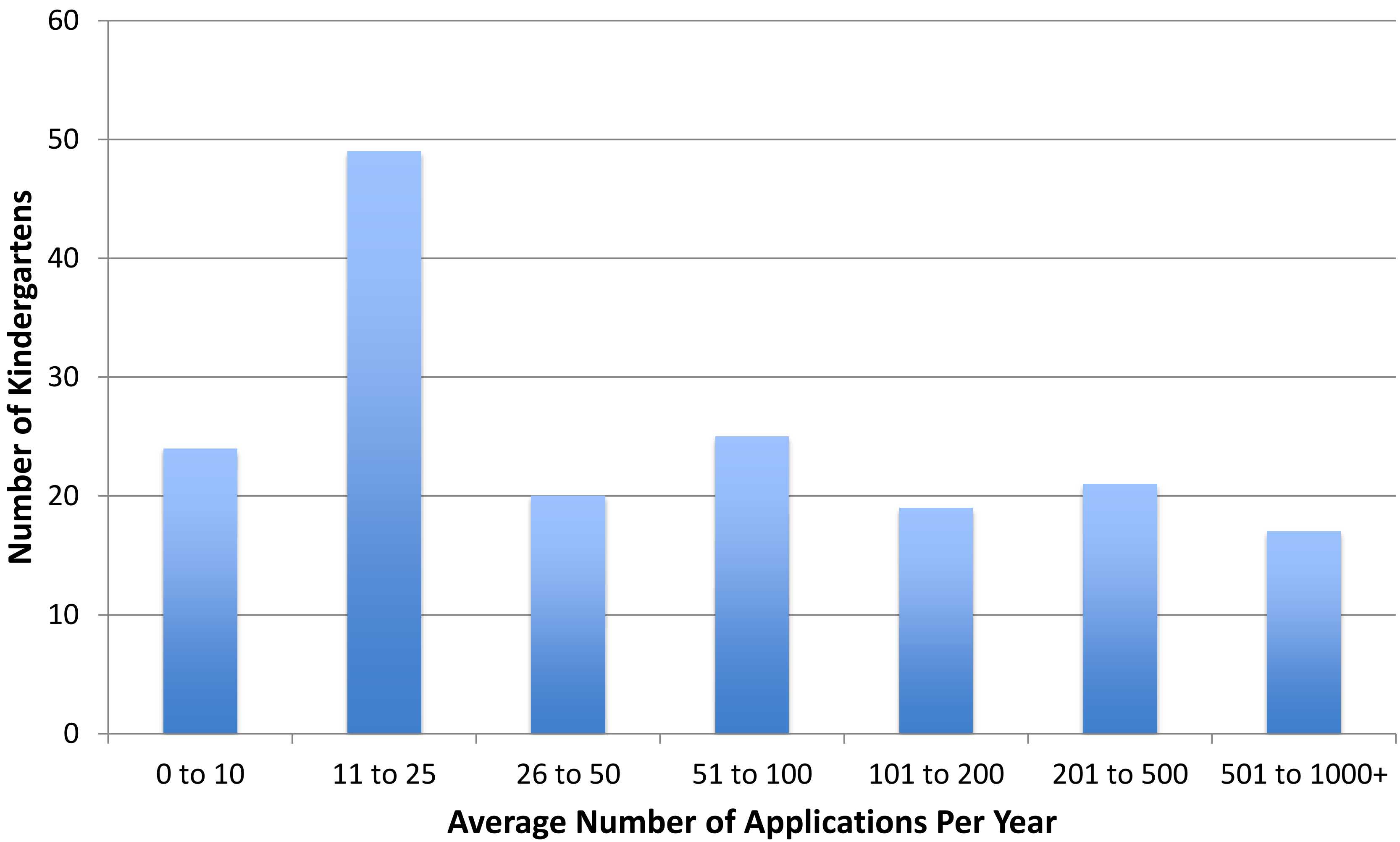
H3: Matches were less efficient after 2009-10;

H4: Diversity of enrollment increased.

Number of Participants Per Year



Distribution of Kindergarten Apps

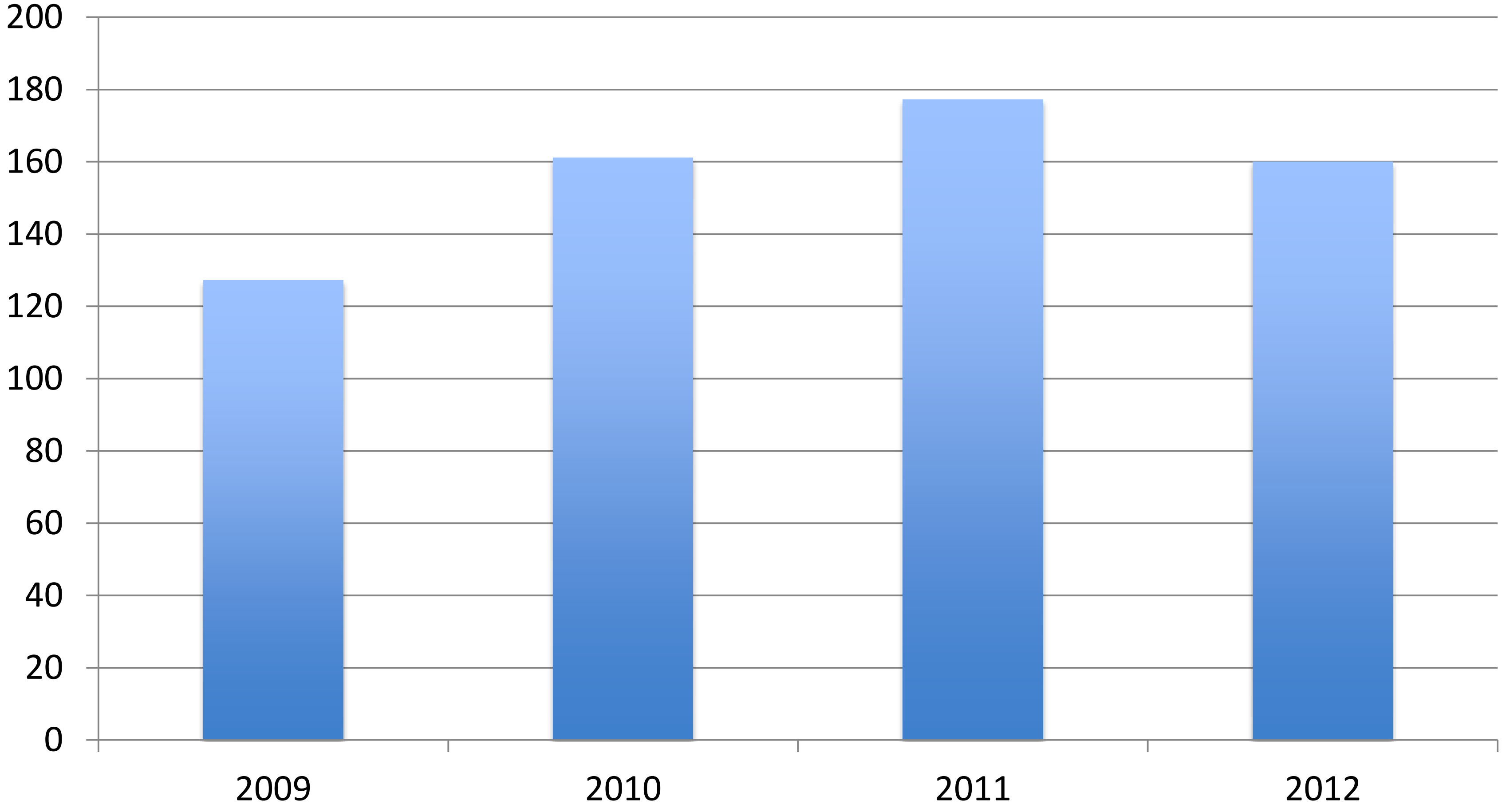


H1: Applications Received

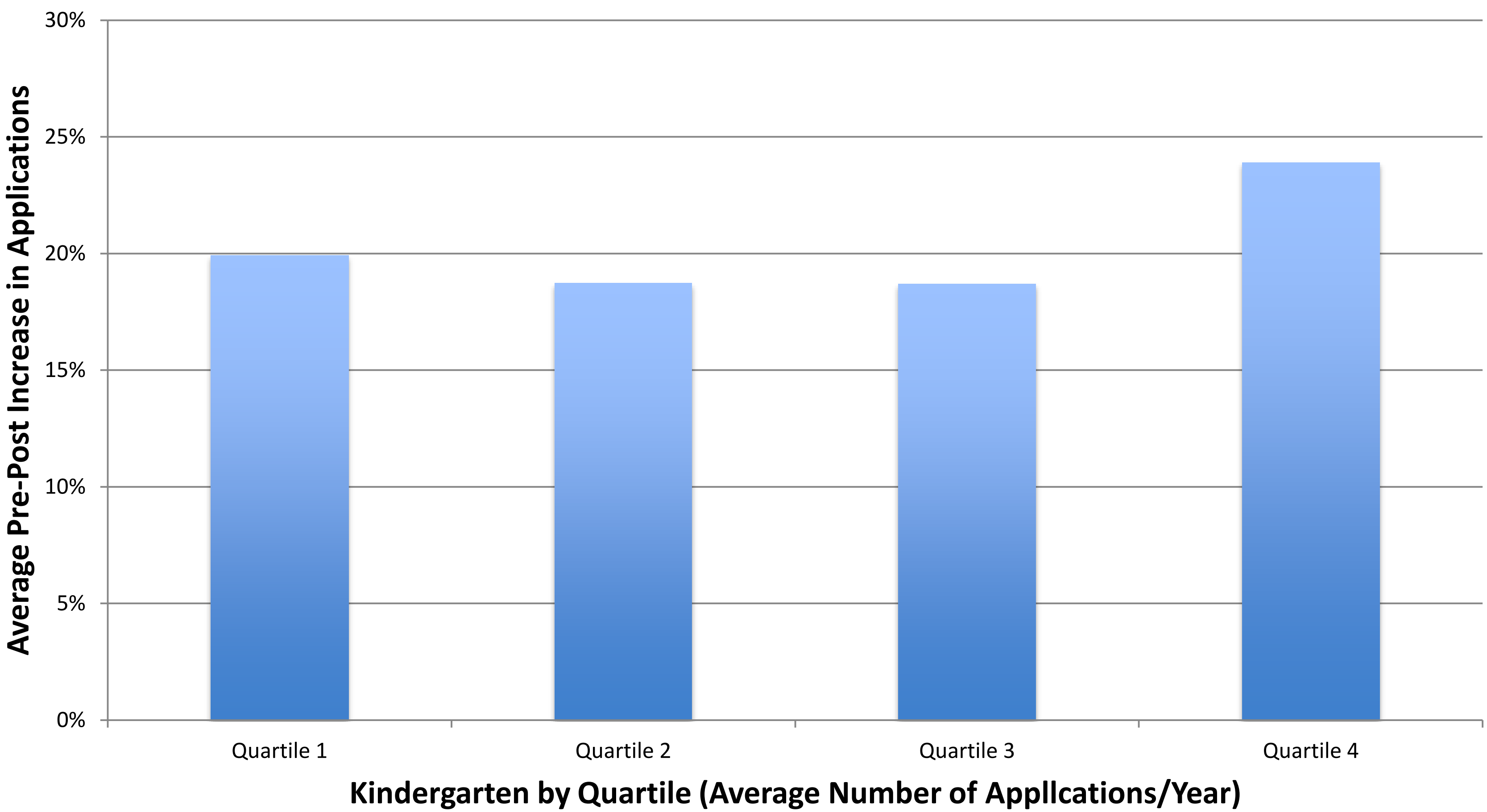
We use paired t-tests and regression analysis to compare the number of applications per school before and after the adoption of Common App.

We limit analysis to the 175 kindergartens that reported applications each year in the sample.

Average Applications Per Year



Change in Applications by Kindergarten Quartile

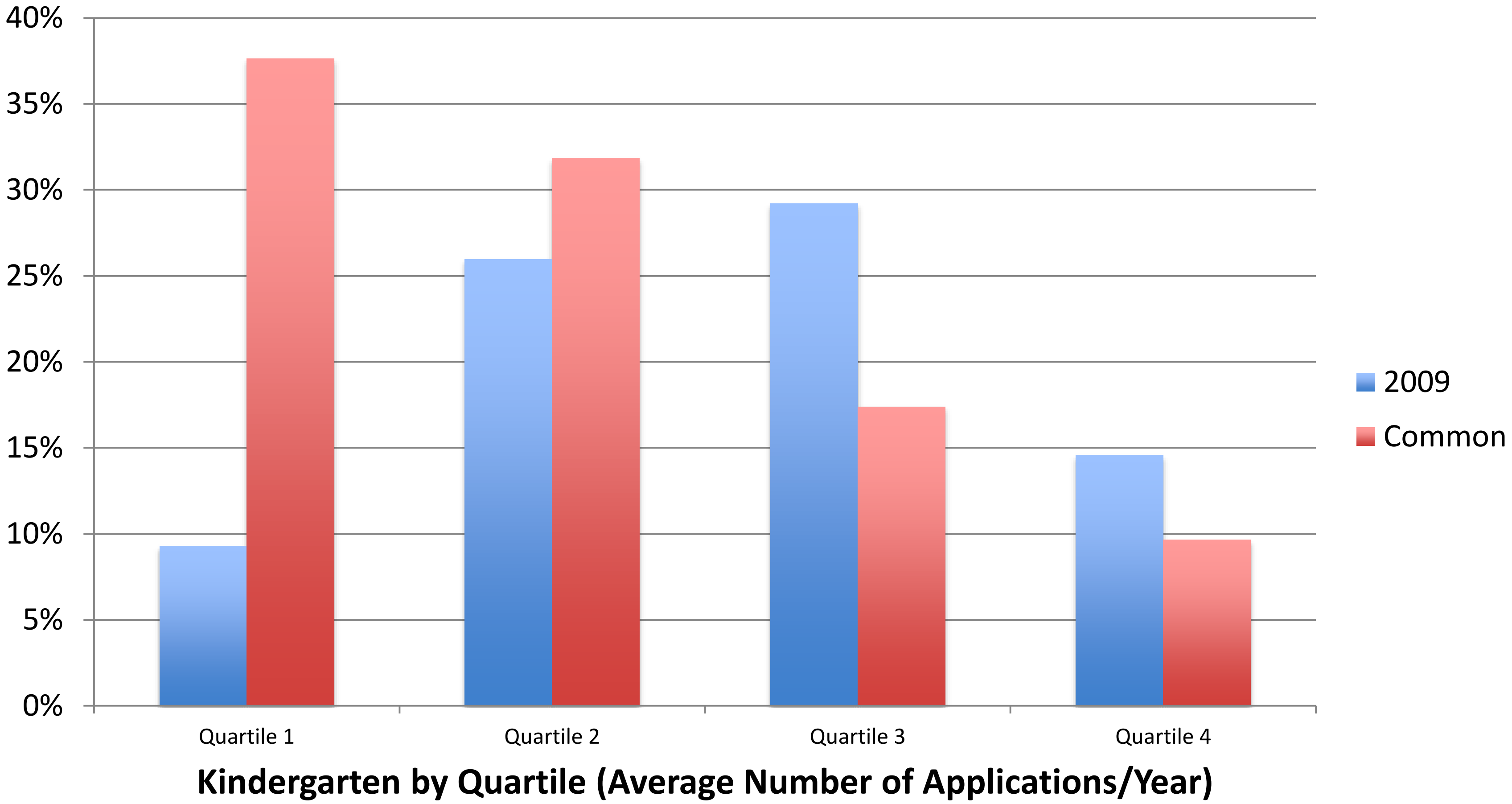


H2: Change in Admissions Rate

We use paired t-tests and regression analysis to compare the average admission rate per school before and after the adoption of Common App.

The results are sensitive to the inclusion/exclusion of Quartile 1 and 2 schools in the distribution of applications per year.

Admissions Rate by Kindergarten Quartile



H3: FOSD and Matches

We simulated the distribution of ordinal rank matches (1st choice, 2nd choice …) before and after the Common Application assuming

A. IID distribution of utility values for each school where a student applies.

B. Identical rank order of schools for all according to number of apps by school.

Each yields similar results.

H4: Diversity of Enrollment

- We see little to no change in diversity of either the applicant pool or enrolling students after the introduction of the Common Application.

- Each group appears to have increased its rate of application by similar proportion from 2009-2010 to 2011-2013.

Conclusion

The Common Application may be better suited to college admission than to K-12 assignments.

In theory, it produces worse matches than a decentralized scheme or Random Priority matching.

We find strong support for the predictions of the theoretical model for Chicago High Schools and weaker support for Chicago Kindergartens.